Math 1320
C. Mundy-Castle
Fall '14

NAME KEY

FINAL EXAM

Thursday, December 11th, 2014

#1	/10
#2	/10
#3	/10
#4	/10
#5	/10
#6	/10
#7	/10
#8	/10
#9	/10
#10	/10
Total	/100

You may use all of your previous formula sheets (or a new formula sheet) on this exam. **Show all of your work to receive full/partial credit!**

1) Given $f(x) = 10 - 2x - 5x^2$, find

a) $f(-1) = 10 - 2(-1) - 5(-1)^2 = 10 + 2 - 5 = 7$

b) $f(w) = 10 - 2w - 5w^2$

c) $f(x+h) = 10 - 2(x+h) - 5(x+h)^2$

$= 10 - 2x - 2h - 5x^2 - 10xh - 5h^2$

2) A battery manufacturer can produce 10,000 batteries per day at a total cost of \$20,000 and 22,000 batteries at a total cost of \$32,000. Find the manufacturer's daily fixed costs and marginal cost per battery.

$C(x) = mx + b$ $\quad\quad$ $m = \dfrac{32000 - 20000}{22000 - 10000} = \dfrac{12000}{12000} = 1$

$C(x) = x + b \rightarrow 20000 = 10000 + b \rightarrow b = 10000$

$C(x) = x + 10000$

Fixed cost: \$10,000

Marginal cost: \$1/battery

3) The Hell's Angels are planning their annual fund-raising drive. They plan to sell cookies, and will charge \$1.55 per cookie. The only expenses they will incur are the cost of the cookie dough, estimated at 30¢ per cookie, and the \$500 charge for renting a booth at the local Renaissance Fair.

a) Write down the associated cost, revenue, and profit functions.

$C(x) = 0.30x + 500$

$R(x) = 1.55x$

$P(x) = 1.55x - (0.30x + 500)$

$= 1.25x - 500$

b) How many cookies must the Hell's Angels sell in order to break even?

$1.25x - 500 = 0 \rightarrow 1.25x = 500$

$x = 400$

400 cookies to break even

c) What profit (or loss) results from the sale of 400 cookies?

$P(400) = 1.25(400) - 500 = 0$

No profit, No loss

4) The cost function for a company to produce a particular item is $C(x) = -8x + 90$, where x is the price (in dollars) the company charges for the item. The revenue function for the item is $R(x) = -10x^2 + 100x$. Find the daily profit as a function of x and determine the unit price you should charge to obtain the largest possible daily profit. What is the largest possible daily profit?

$$P(x) = -10x^2 + 100x - (-8x + 90)$$

$$= -10x^2 + 108x - 90$$

$$X_{max} = \frac{-108}{2(-10)} = \$5.40$$

charge \$5.40 to maximize profit

$$P(5.40) = -10(5.40)^2 + 108(5.40) - 90 = \$201.6$$

Max profit is \$201.60

5) A bacteria culture starts with 500 bacteria. Three hours later there are 800 bacteria. Find an exponential model for the size of the culture as a function of time t in hours, and use the model to predict how many bacteria there will be after 24 hours.

$y = Ab^t \rightarrow y = 500b^t$

$800 = 500b^3 \rightarrow \frac{8}{5} = b^3 \rightarrow b = \left(\frac{8}{5}\right)^{1/3} \approx 1.1696$

$y = 500(1.1696)^t$

After 24 hrs: $y = 500(1.1696)^{24} \approx 21{,}472$

6) A test requires that you answer first Part A and then either Part B or Part C. Part A consists of five true-false questions, Part B consists of four multiple-choice questions with one correct answer out of five, and Part C consists of three questions with one correct answer out of four. How many different completed answer sheets are possible?

$2^5 \cdot 5^4 + 2^5 \cdot 4^3 = 22{,}048$

7) The half-life of a particular element is 24 years.

a) Obtain an exponential model for the element in the form $Q(t) = Q_0 e^{-kt}$. (Round coefficients to three significant digits.)

$$k = \frac{\ln\left(\frac{1}{2}\right)}{24} \simeq -0.0289$$

$$Q(t) = Q_0 e^{-0.0289t}$$

b) Use your model to predict, to the nearest year, the time it takes for one-fourth of the sample of the element to decay.

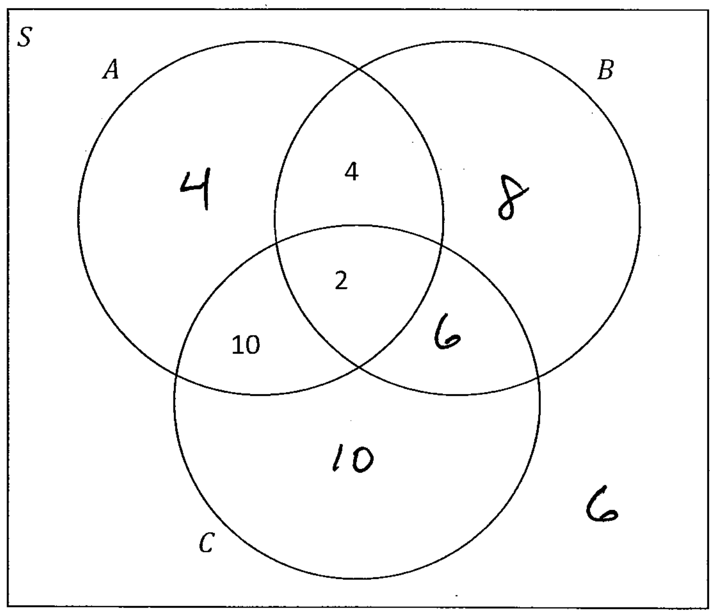
$$\frac{3}{4}\cancel{Q_0} = \cancel{Q_0} e^{-0.0289t}$$

$$\frac{3}{4} = e^{-0.0289t} \rightarrow \ln\left(\frac{3}{4}\right) = -0.0289\,t$$

$$t = \frac{\ln(3/4)}{-0.0289} \simeq 9.954$$

about 10 years.

8) Use the given information to complete the solution of the partially solved Venn diagram.



$n(A) = 20, n(B) = 20, n(C) = 28, n(B \cap C) = 8, n(S) = 50$

9) Tom has just received won a lottery worth $500,000. He decides to take the annuity option that guarantees equal monthly payments over 20 years. The annuity earns 5.2% interest, compounded monthly. How much will his payments need to be so that the $500,000 draws down to zero after 20 years?

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = 500000 \frac{\frac{0.052}{12}}{1-\left(1+\frac{0.052}{12}\right)^{-(12)(20)}}$$

$$= \$3355.27$$

10) Sven is given a bag containing 5 red marbles, 4 green ones, 3 white ones, and 2 purple ones. He grabs six of them. Find the probabilities of the following events, expressing each as a fraction in lowest terms.

$n(S) = C(14,6) = 3003$

a) He has all the red ones.

$n(E) = C(5,5) \cdot C(9,1) = 9$ $\quad P(E) = \frac{9}{3003} = \frac{3}{1001} \quad (\approx 0.003)$

b) He has at least two white ones.

$n(E) = C(3,2) \cdot C(11,4) + C(3,3) \cdot C(11,3) = 1155, \quad P(E) = \frac{1155}{3003} = \frac{5}{13} \quad (\approx 0.385)$

c) He has three green ones and one of each of the other colors.

$n(E) = C(4,3) \cdot C(5,1) \cdot C(3,1) \cdot C(2,1) = 120$

$P(E) = \frac{120}{3003} = \frac{40}{1001} \quad (\approx 0.04)$

d) He has no red ones.

$n(E) = C(9,6) = 84, \quad P(E) = \frac{84}{3003} = \frac{4}{143} \quad (\approx 0.028)$